General Physics 1 (AnPh111v)

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Kinematics in Cartesian Coordinates

Outline

- Main Concepts in Kinematics

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- Kinematics in Cartesian Coordinates
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Part I

Main Concepts in Kinematics

Kinematics

Kinematics

Kinematics is a subfield of physics, developed in classical mechanics, that describes the geometric properties of the mechanical motion of material point(s), bodies (objects), and systems of bodies (groups of objects) without considering the mass and forces that cause them to move.

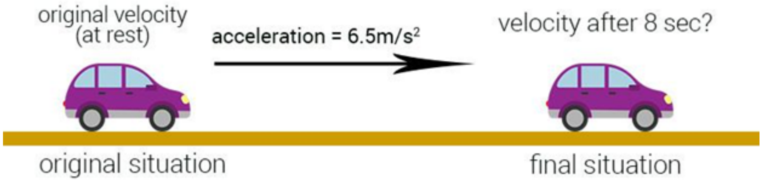
Material Point:

A body which dimensions can be neglected under given conditions!

Kinematics

What we do:

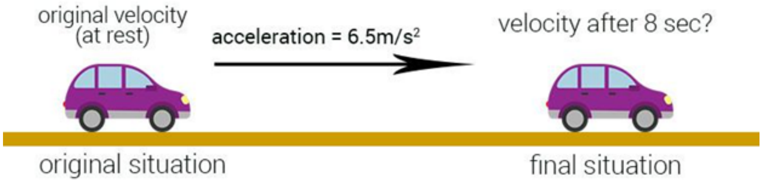
A kinematics problem begins by describing the geometry of the system and declaring the initial conditions of any known values of position, velocity and/or acceleration of the material point or points within the system.



Kinematics

What we do:

Using arguments from geometry, the position, velocity and acceleration of any unknown parts of the system can be determined as a function of time, a distance can be calculated etc.

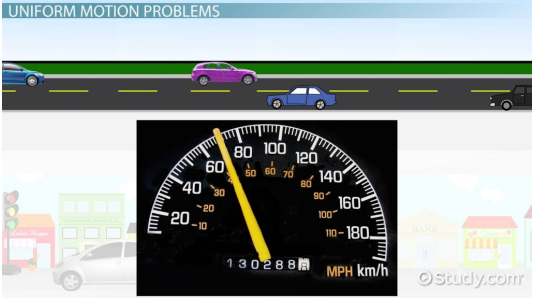


Motion

- Rectilinear Motion: straight line trajectory.
- Curvilinear Motion: curved line trajectory (next chapter).
- The point motion is described as a function of time!

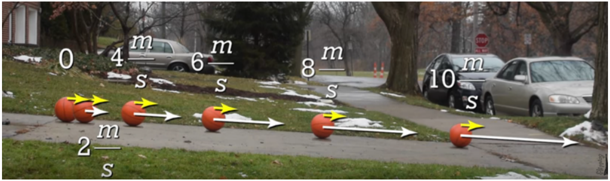
Rectilinear Motion

- Straightforward uniform motion:
  - If the direction of the velocity vector does not change, then the motion is called straightforward. If the speed module does not change over time, the motion is called uniform. Distance: $s = v(t - t_0)$, position: $x(t) = x_0 + v(t - t_0)$.



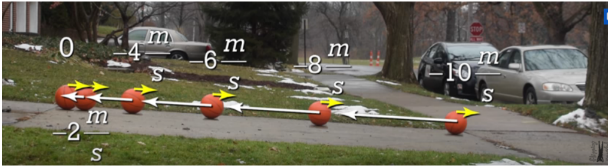
Rectilinear Motion

- Accelerated movement:
  - If the acceleration vector is constant in the same direction of the velocity vector.
  - $s = \frac{v^2 - v_0^2}{2a}$, $x(t) = v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2$



Rectilinear Motion

- Slower movement:
  - If the acceleration vector is constant and is opposed to the velocity vector.
  - $v(t) = v_0 + a(t - t_0)$



Rotational Motion

Rotational motion is a movement in which all points of an absolutely rigid body move in circles whose centers lie on one straight line (i.e rotation axis).

- Point velocity: $v = \omega R$, $T = \frac{2\pi}{\omega}$
- Normal acceleration: $a_N = \frac{v^2}{R} = \omega^2 R$
- Frenet Acceleration: $\vec{a} = \vec{a}_T + \vec{a}_N$
- To be improved during this course

Part II

Kinematics in Cartesian Coordinates

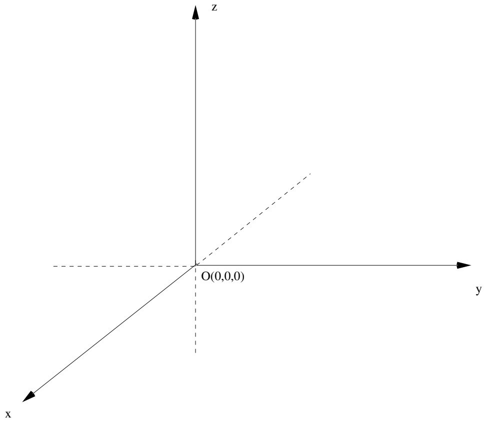
Cartesian Coordinate System

Definition

A Cartesian coordinate system is a coordinate system that specifies each point uniquely in a plane or in space by a set of numerical coordinates, which are the signed distances to the point from two fixed perpendicular oriented lines, measured in the same unit of length.

Each reference line is called a coordinate axis or just axis (plural axes) of the system, and the point where they meet is its origin, at ordered pair O(0,0,0). The coordinates can also be defined as the positions of the perpendicular projections of the point onto the two or three axes, expressed as signed distances from the origin.

Cartesian Coordinate System

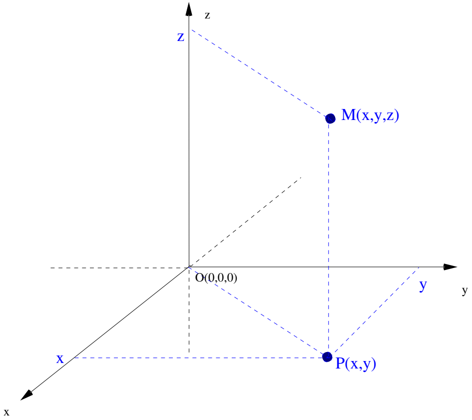


Point Coordinates

Coordinates

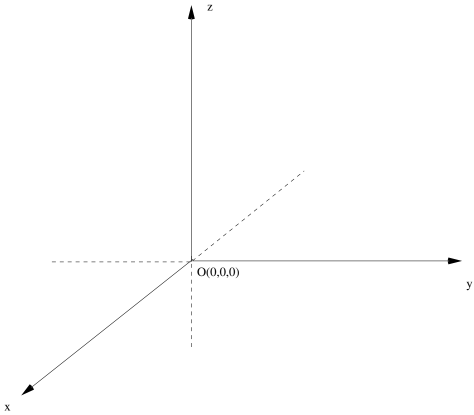
The coordinates of any point on $\mathbb{R}^3$ are written in the form: M(x,y,z) where x, y and z are measured in the same unit of length.

Point Coordinates



Unit Vectors

- Unit vector along the x-axis:
$$\vec{u}_x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \text{ or } (1,0,0)$$
- Unit vector along the y-axis:
$$\vec{u}_y = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ or } (0,1,0)$$
- Unit vector along the z-axis:
$$\vec{u}_z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \text{ or } (0,0,1)$$



Unit Vectors

- Unit vector along the x-axis:

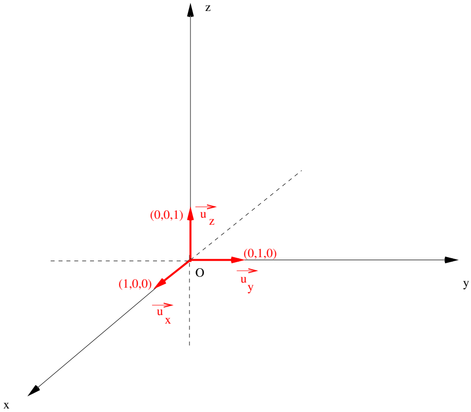
$$\vec{u}_x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \text{ or } (1, 0, 0)$$

- Unit vector along the y-axis:

$$\vec{u}_y = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ or } (0, 1, 0)$$

- Unit vector along the z-axis:

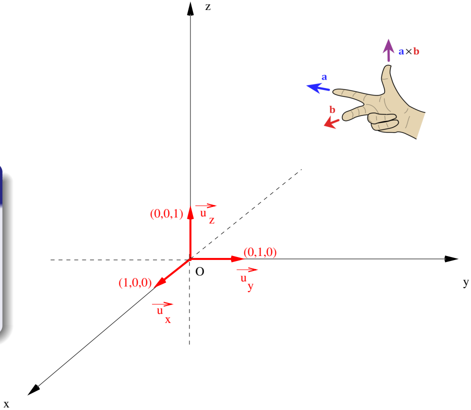
$$\vec{u}_z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \text{ or } (0, 0, 1)$$



Unit Vectors

Exercise:

Check the unit vectors combine to a direct trihedron: $\vec{u}_z = \vec{u}_x \wedge \vec{u}_y$ & all permutations (right-hand rule).



Frame of Reference

Galilean Frame of Reference

In physics, a frame of reference (or reference frame) consists of an abstract coordinate system and the set of physical reference points that uniquely fix (locate and orient) the coordinate system and standardize “time" measurements within that frame: coordinate system with time measurement.

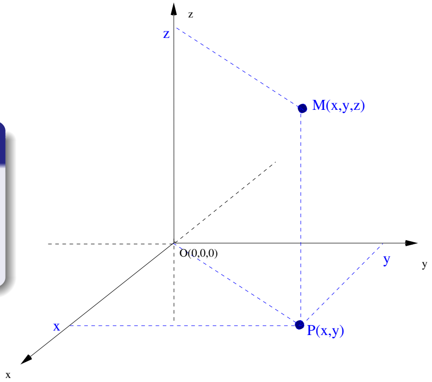
Frame of Reference

Choice of the reference frame

The choice of the reference frame should be done according to the system geometry. In practice, the origin is fixed such that any point position can be given as a function of time regarless of the type of the coordinate system.

Cartesian Coordinates

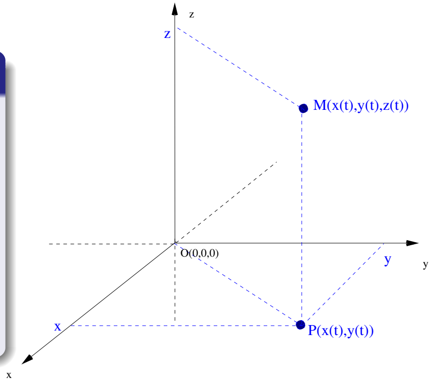
The M point position is determined by the lengths: x, y & z.



Cartesian Reference Frame

The M point time dependent position is determined by the scalar functions:

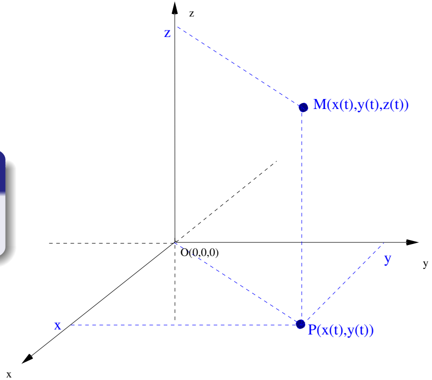
$$t \mapsto x(t), t \mapsto y(t) \;\&\; t \mapsto z(t)$$



Position Vector or Radius Vector

Position Vector

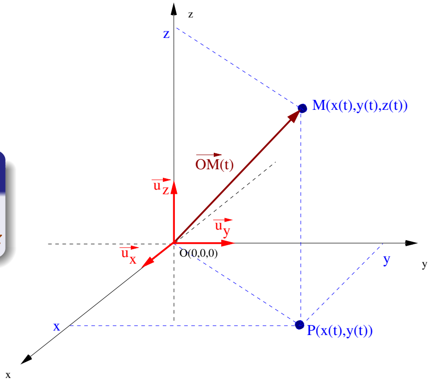
$$\overrightarrow{OM}(t) = (x(t), y(t), z(t))$$



Position Vector or Radius Vector

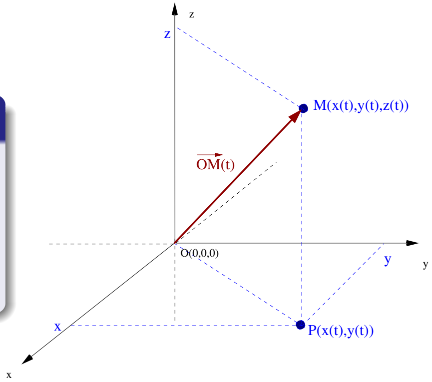
Position Vector

$$\overrightarrow{OM}(t) = x(t)\vec{u}_x + y(t)\vec{u}_y + z(t)\vec{u}_z$$



In practice:

$$\overrightarrow{OM}(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} \in \mathbb{R}^3$$



Point Trajectory

Trajectory

The trajectory, also named path, can be regarded as the track left by the point during the motion. The choice of the reference frame should preferably match the shape of the trajectory.

- The parametric (or coordinate time dependent) equations on $\mathbb{R}^3$ is given by the map $\Gamma:\mathbb{R} \mapsto \mathbb{R}^3$:

$$\Gamma:t \mapsto (x(t), y(t), z(t)),$$

such that for any measured time it is possible to draw the path left behind by the point.

Point Trajectory

Trajectory

The trajectory, also named path, can be regarded as the track left by the point during the motion. The choice of the reference frame should preferably match the shape of the trajectory.

- The parametric (or coordinate time dependent) equations on $\mathbb{R}^2$:

$$t \mapsto x(t), \quad t \mapsto y(t),$$

where it might become possible to recover the function: $y = f(x)$ on the plane.

Point Trajectory

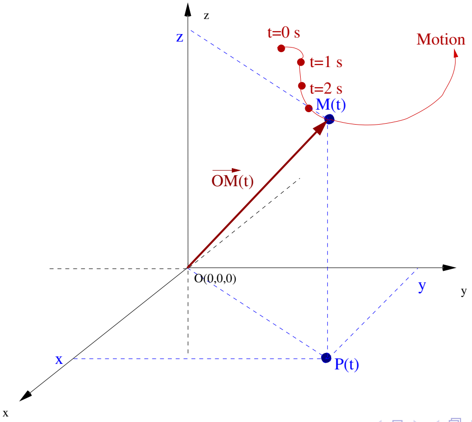
Trajectory

The trajectory, also named path, can be regarded as the track left by the point during the motion. The choice of the reference frame should preferably match the shape of the trajectory.

- Such equations are different in cylindrical and spherical coordinates (see next chapter).

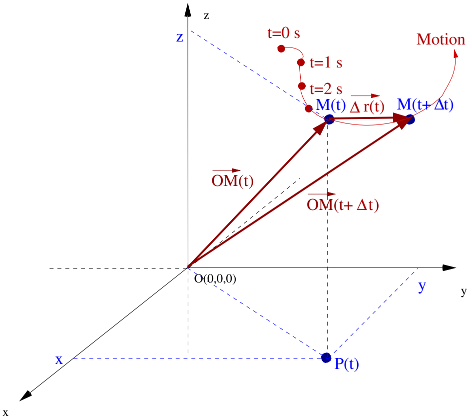
Trajectory

The initial condition at $t = 0$ is also necessary in order to draw the path from the beginning.



Displacement Vector

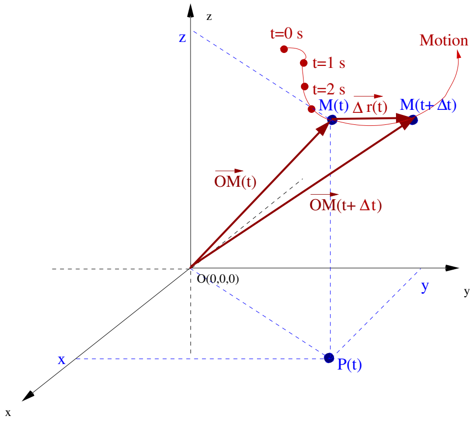
Ch. in the position vector over Δt: $\overrightarrow{\Delta r}(t) = \overrightarrow{OM}(t+\Delta t) - \overrightarrow{OM}(t)$!



Velocity Vector & Speed

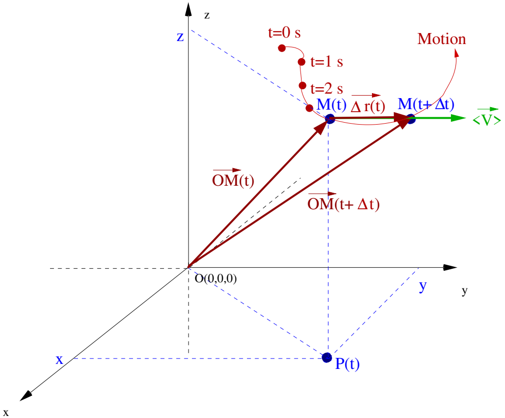
Distance

And $|\overrightarrow{\Delta r}|$ is the displacement over the time interval Δt.



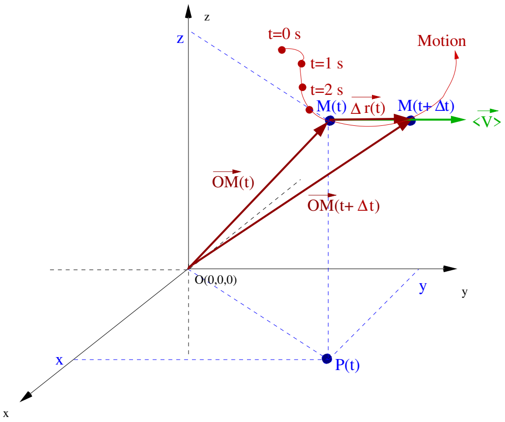
Average Velocity Vector

Given by the ratio of the displacement vector $\overrightarrow{\Delta r}$ and Δt: $\langle \vec{V} \rangle = \frac{\overrightarrow{\Delta r}}{\Delta t}$



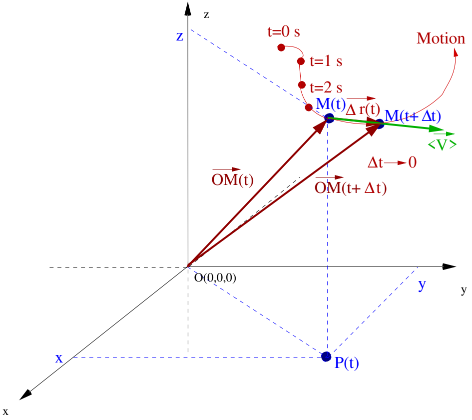
Velocity Vector

As $\Delta t \rightarrow 0$, $\langle\vec{V}\rangle$ becomes the time derivative of the position vector.



Velocity Vector

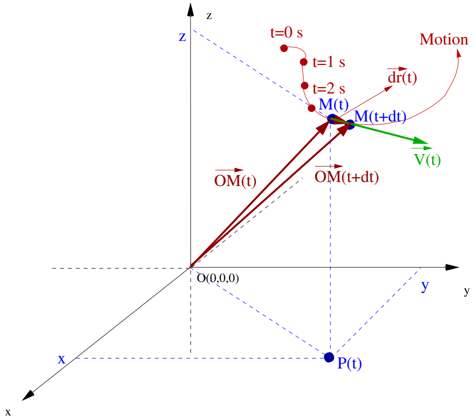
As $\Delta t \to 0$, $\langle \vec{V} \rangle$ becomes the time derivative of the position vector.



Velocity Vector & Speed

Velocity Vector

$$\vec{V}(t) = \lim_{\Delta t \to 0} \langle \vec{V}(t) \rangle = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta r}}{\Delta t} = \frac{d\vec{r}}{dt} = \dot{x}(t)\vec{u}_x + \dot{y}(t)\vec{u}_y + \dot{z}(t)\vec{u}_z$$



Velocity Vector & Speed

- (Exercise) The velocity vector is tangent to the path and can be rewritten in the equivalent form:

$$\vec{V}(t) = \begin{pmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{z}(t) \end{pmatrix}$$

- Since $\vec{V}(t) \in \mathbb{R}^3$ and the basis is orthonormal, its norm reads

$$V(t) = \|\vec{V}(t)\| = \pm\sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} \quad \textbf{(algebraic value)}$$

- The speed of an object is the magnitude $|\vec{V}|$ of its velocity. It is a scalar quantity:

$$|V| = \frac{ds}{dt}$$

where ds is a very small displacement. For instance, one refers to the speed of sound, speed of light etc.

Velocity Vector & Speed

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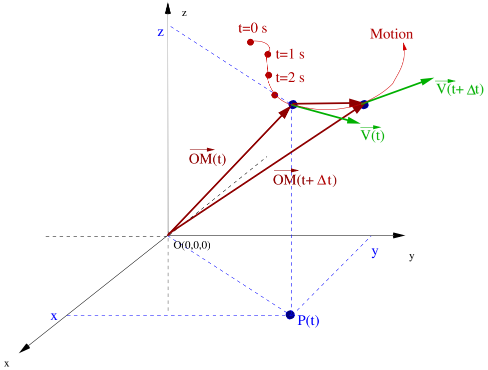
- If a particle is moving along the x-axis at +7 m/s and another particle is moving along the same axis at -7 m/s, they have different velocities, but both have the same speed of 7 m/s.

Average Acceleration Vector

The average acceleration of a particle over a time interval is defined as the ratio: $\langle\vec{a}(t)\rangle = \frac{\Delta\vec{V}}{\Delta t} = \frac{\vec{V}(t+\Delta t)-\vec{V}(t)}{\Delta t}$.



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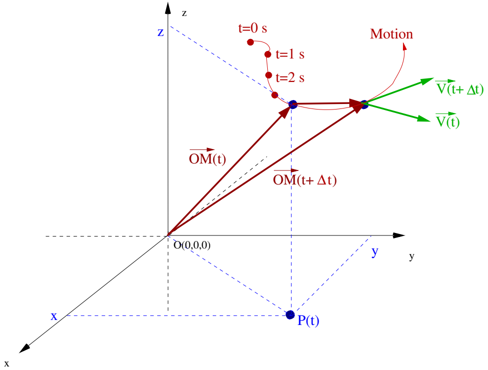

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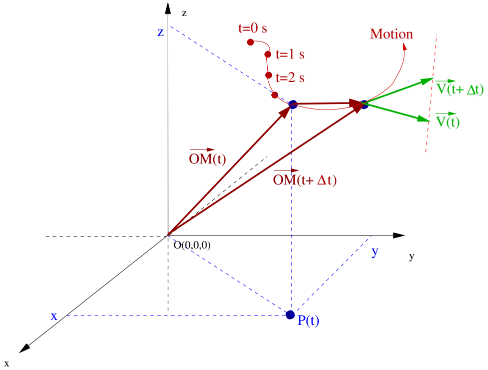

Acceleration Vector

Average Acceleration Vector

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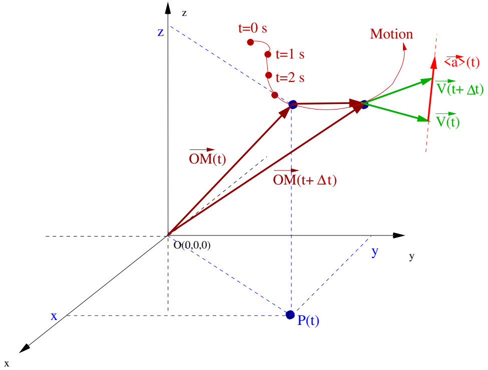

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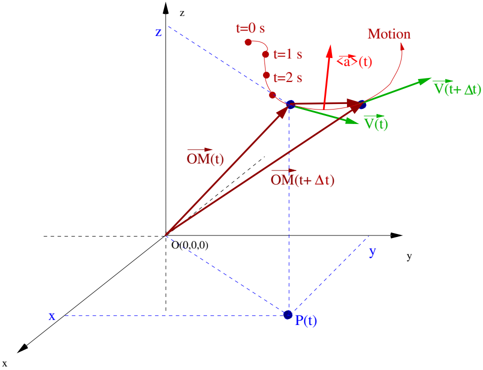
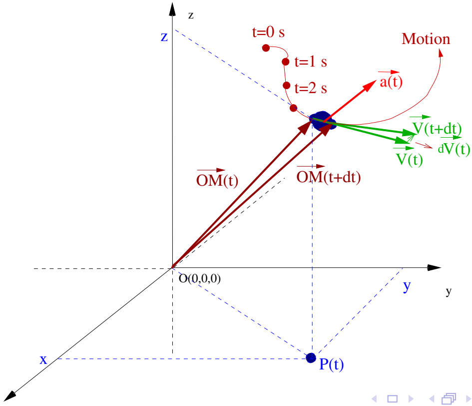


Acceleration Vector

Average Acceleration Vector

The acceleration of the particle is the limit of the average acceleration: $\vec{a}(t) = \lim_{\Delta t \to 0} \frac{\Delta \vec{V}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\vec{V}(t+\Delta t) - \vec{V}(t)}{\Delta t} = \frac{d\vec{V}}{dt}$.



Acceleration Vector

- (Exercise) The acceleration vector can be rewritten in the equivalent forms:

$$\vec{a}(t) = \ddot{x}(t)\vec{u}_x + \ddot{y}(t)\vec{u}_y + \ddot{z}(t)\vec{u}_z = \begin{pmatrix} \ddot{x}(t) \\ \ddot{y}(t) \\ \ddot{z}(t) \end{pmatrix}$$

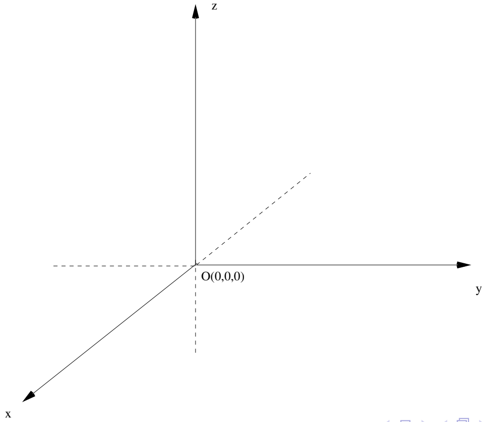
- Since $\vec{a}(t) \in \mathbb{R}^3$ and the basis is orthonormal, its norm reads

$$a(t) = \|\vec{a}(t)\| = \sqrt{\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2}$$

Length Element in Cartesian Coordinates

Length Element

In geometry, the line element or length element can be may be thought of as a differential arc length, is a is denoted by $d\vec{\ell}$.



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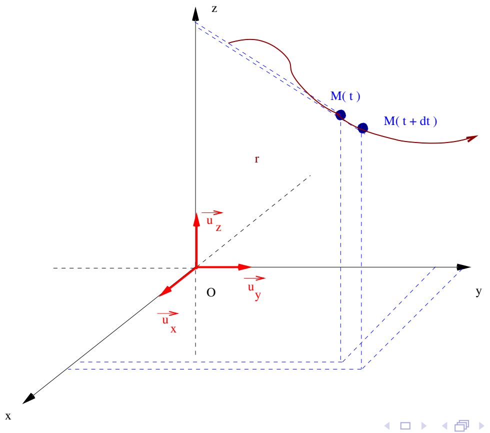

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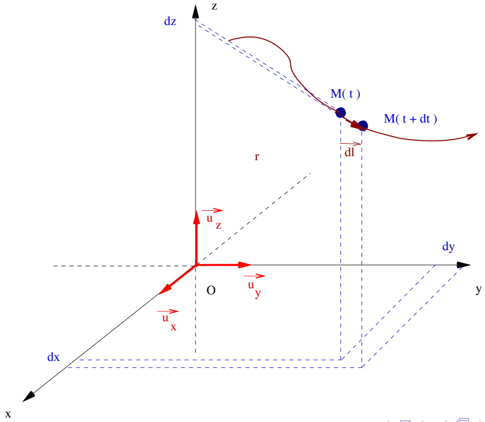
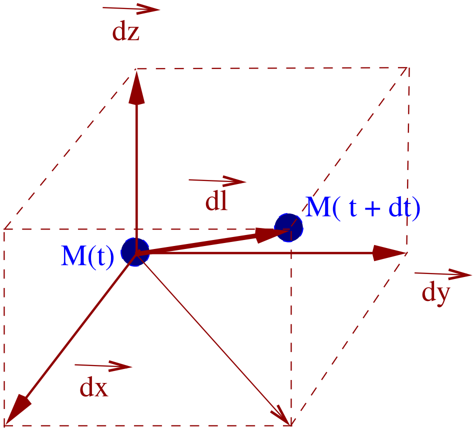
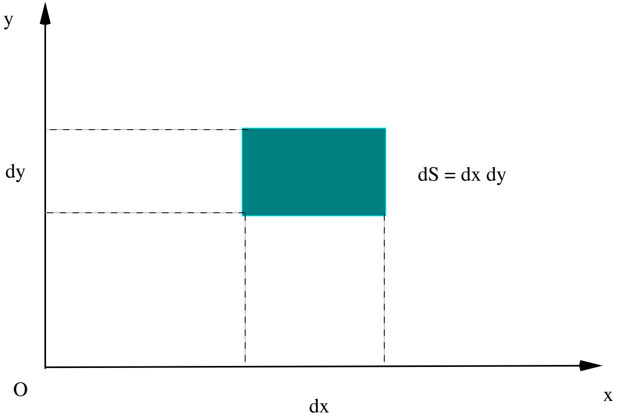


Length Element

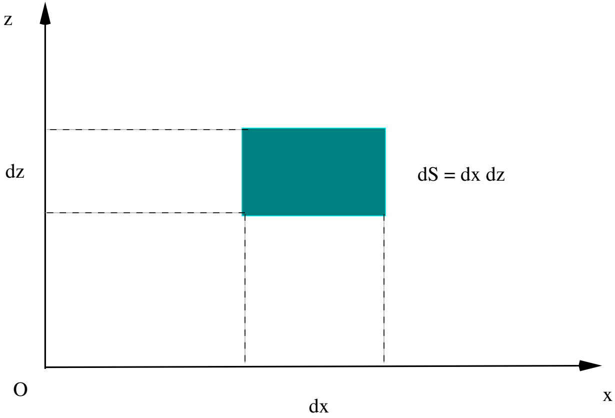
Off the figure: $d\vec{\ell} = \vec{dx} + \vec{dy} + \vec{dz} = dx\vec{u}_x + dy\vec{u}_y + dz\vec{u}_z$.



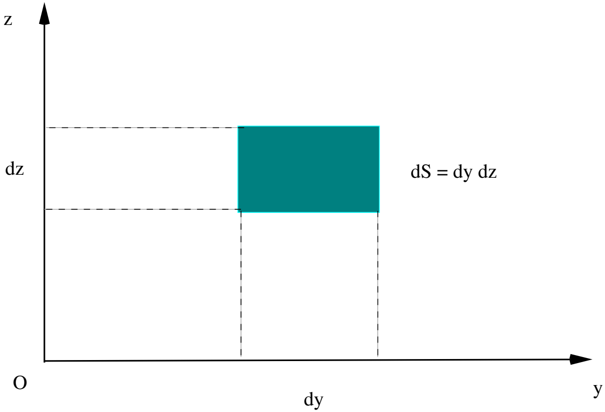
Area Element in Catersian Coordinates



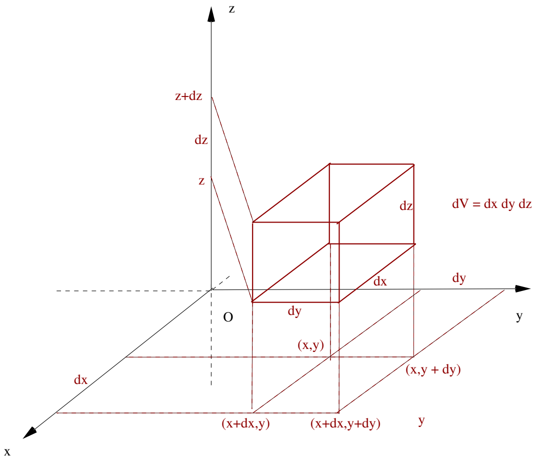
Area Element in Catersian Coordinates

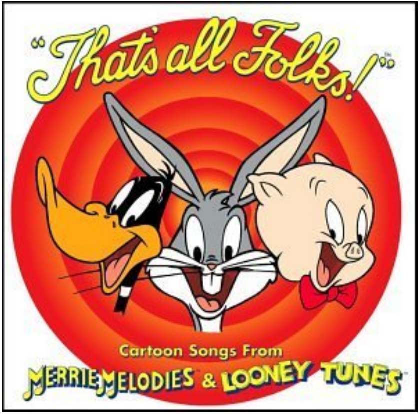


Area Element in Catersian Coordinates



Volume Element in Catersian Coordinates




"That's all Folks!"
Cartoon Songs From
MERRIE MELODIES & LOONEY TUNES